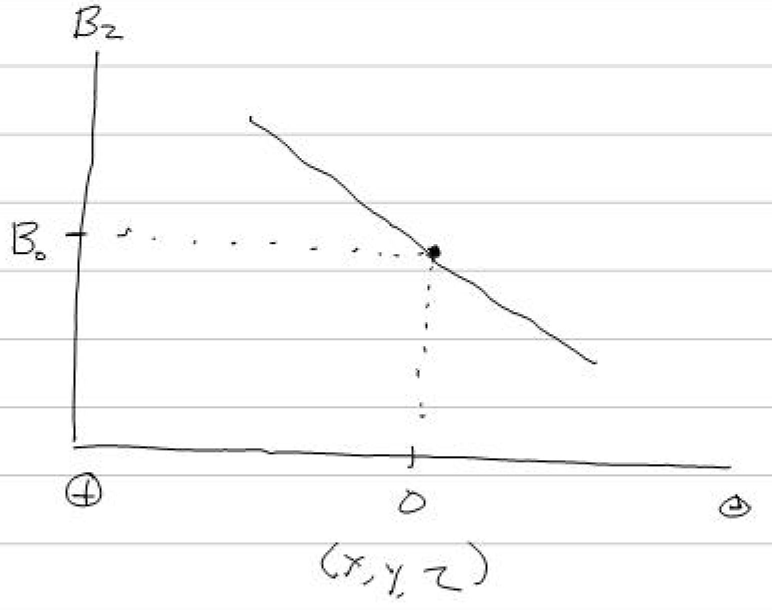
Review

Spatial Encoding $\Rightarrow$ Apply Magnetic Field Gradients

$$\omega = \gamma B_z \; ; \quad G_{x,y,z} = \frac{\Delta B_z}{\Delta_{x,y,z}} = \text{constant}$$

$= $ linear change in B_z

$$B_z = B_0 + \underbrace{G_{x,y,z} \cdot (x,y,z)}_{\text{this is} \ll B_0}$$



Easy to change slope / direction.

$$\omega(q,t) = \underbrace{\gamma B_0}_{\omega_0} + \underbrace{\gamma \cdot G_{x,y,z}(t) \cdot q}_{\omega_G(q,t)}$$

ω_0: Not a fn of position or time

$\omega_G(q,t)$: linearly dependent on position, "Frequency encoding"

$$\vec{G}(t) = \vec{G}_x(t)\,\hat{i} + G_y(t)\,\hat{j} + G_z(t)\,\hat{k}$$

can describe any gradient direction

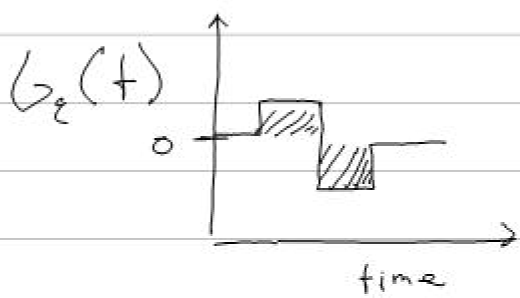
$$\varphi_G(z,t) = \int_0^t \omega(z,t')\,dt'$$

$$= -\gamma \cdot z \int_0^t G(t')\,dt'$$

If G is not a function of time

$$= -\gamma \cdot z \cdot G \cdot t$$

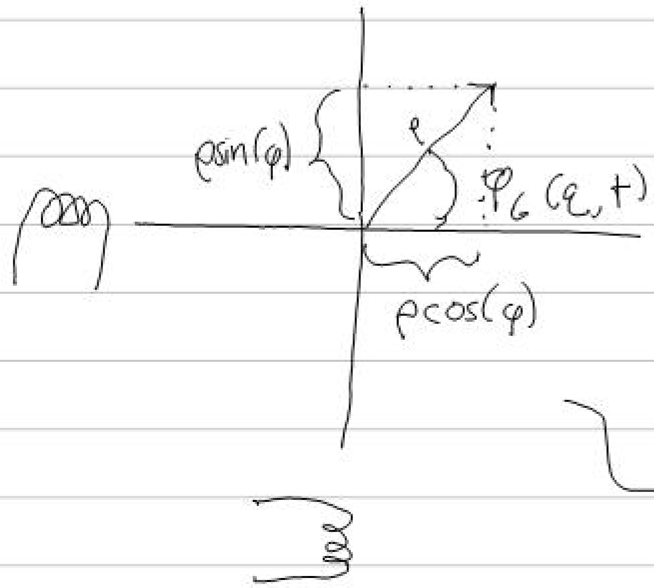
So, phase is linear in position,
"phase encode"



Area under gradient that matters

Consider a small volume element inside our object / sample.

At some time after exposure to G, ~~sins~~ spins have $\varphi_G(z,t)$



Transverse Plane
(M_{xy})

$\rho(\xi)$ = spin density $\therefore M_{xy}$

$$e^{i\varphi} = \cos(\varphi) + i\sin(\varphi)$$

Good News

① Phase Sensitive
② Detect all volume elements at the same time

Bad News

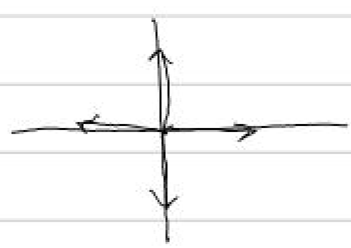
① Detect sum of the phases.

$$S_{signal}(t) = \int_{\xi} \underbrace{\rho(\xi)}_{\text{spin density}} \cdot e^{i\varphi_G(\xi, t)}\, d\xi$$

temporal frequency

Ex. $x + y + z = 20$ But what is each?

Phases:
4 volume element



If no signal is detected, there are multiple ways the spins could be aligned.

Spatial Frequencies

$\omega = \gamma B$ $\qquad\qquad \gamma = \frac{\omega}{B} = \frac{rad}{s \cdot gauss}$

temporal frequency, $\frac{radians}{sec}$

$\frac{\omega}{2\pi} = Hz$

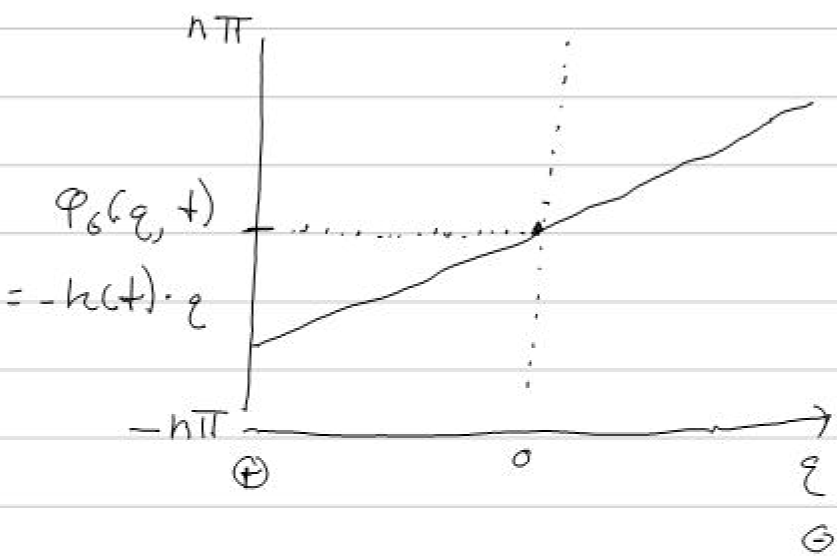
$k(t) = \gamma \int_0^t G(t')\,dt'$

Suppose G is not a function of time.

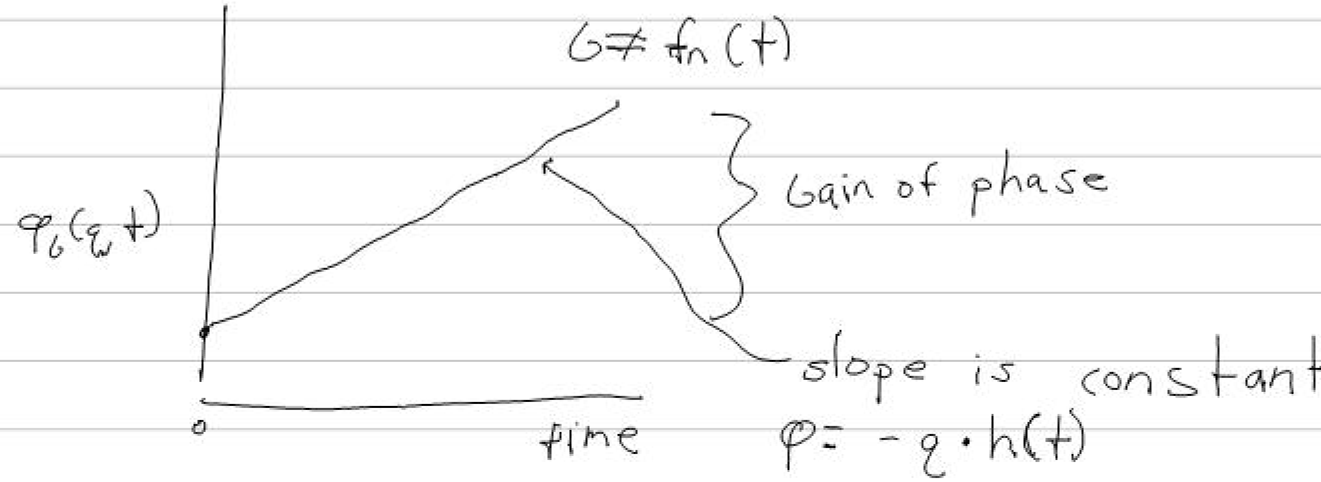
$k = \gamma \cdot G \cdot t = \frac{rad}{s \cdot gauss} \cdot \frac{gauss}{cm} \cdot s = \frac{rad}{cm}$

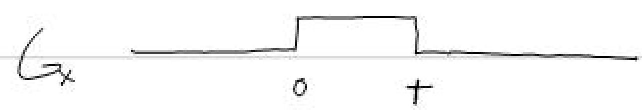
k space

Can get $\frac{cycles}{cm}$ by dividing by 2π.



more phase change, tighter pitch
Greater G or longer t.

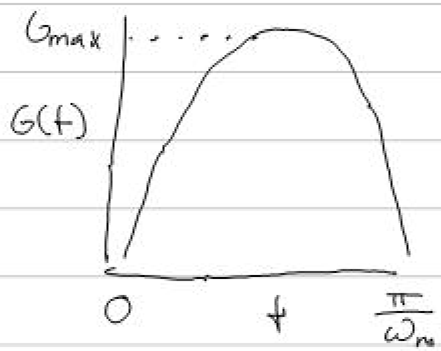




We often want to use a shaped gradient.

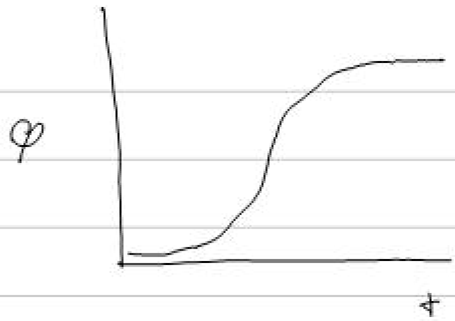
Suppose we sinusoidally modulate G.

$$\cdot\ G(t) = G_{max} \cdot \sin(\omega_m \cdot t)$$



$$\varphi_G(q, t) = -\int_0^{t = \pi/\omega_m} \gamma q\, G(t')\, dt'$$

$$= q \cdot h(t)$$



? May be wrong.

More commonly, $k(t) = \frac{\gamma}{2\pi} \int_0^t G(t')\,dt'$

$\frac{\text{cycles}}{\text{cm}}$

$\varphi_G(t) = -2\pi \cdot q \cdot k(t)$

radians

—

$$S(t) = \int_q \rho(q)\, e^{-i\underbrace{\varphi_G(q,t)}_{-2\pi \cdot q \cdot k(t)}}\, dq$$

$$S[k(t)] = \int_q \rho(q)\, e^{-2\pi q\, k(t)}\, dq$$

Image

⇒ Sum of spatial frequencies

$\underbrace{S[k(t)] + \rho(q)}_{\text{are fourier pairs}}$

$$\rho(q) = \int_k S(k)\, e^{i2\pi k(t) \cdot q}\, dk$$

"spin density" is really M_{xy} at q (T_2, T_1, T_2^*, diffusion, velocity affects this.)